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Technical Paper

Mapping Quantity and Quality: A Foundation for Hybrid Data Modeling in Design

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Mapping Quantity and Quality:

A Foundation for Hybrid Data Modeling in Design

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A scheme for mapping quantity and quality information using threshold concepts and Fuzzy Set Theory is proposed. Model builders can subjectively define the quality space of a system variable with explicit descriptions of distinct thresholds and provide a map to the quantity scale. A mapping mechanism, projecting a quality value to an interval on the quantity scale and identifying the qualitative meaning of a value in quantity, is explained. In addition to some operations on qualitative variables, a fuzzy consolidation function is described. The result is that the model builder's subjective associations with the system variables can be documented so that they can be conveyed to others. The qualitative definitions also make automatic commentary on simulation results possible. This scheme also serves as a foundation for a hybrid modeling and simulation system for analysis and design.

Introduction

A numerical value represents a precise point on a quantitative scale. To be useful, the scale has to be commonly recognized. A measuring unit is often associated with a numerical value so that the magnitude of the value will be less likely to be misunderstood. However, even with a proper measuring unit, the quality of a value can still be misinterpreted because people impose their own subjective associations of quality on the numerical scale. Worse, many design variables are either non-quantifiable or subjectively quantified, causing communication and analysis disasters. Subjectivity, imprecision, and uncertainty in design factors urge the development of a hybrid scheme to combine quantitative and qualitative descriptions and map values from one space to the other.

The scheme presented in this paper involves design variables of ***quantitative***, ***qualitative***, and ***hybrid*** types. Quantitative values are represented by a real number and a measuring unit. A qualitative value is a two-tuple, a ***rank*** and a ***confidence factor*** (***r***, ***cf***). The quality space can be split up into seven regions - three distinct thresholds, two in-betweens, and two outside regions. Each region, or rank, has an adjective and a noun phrase to describe its qualitative meaning. Thus, the system provides three levels of information (with examples describing the weight of a person):

1. A numeric value with its measuring unit, such as ***250 pounds***,

2. An adjective defining the quality, such as ***heavy***, and

3. A noun phrase to refine the definition, such as ***a professional wrestler's weight***.

With these, the annotation system can generate comments like

The weight of the person is ***250 pounds***; it is as ***heavy*** as ***a professional wrestler's weight***.

In a variation on the hybrid type, these regions can be mapped to their numerical counterparts specified by the model builder. The mapping mechanism is based on Fuzzy Set Theory with a few modifications. Numerical values can be mapped to qualitative terms and comments such as those above can be automatically constructed.

Related Work

Modeling a system involves the processes of abstracting attribute variables, measuring their values, formulation the changes associated with time, constructing a network for related variables, and describing the current condition of the system at a specific time. All these processes rely on a scientific procedure -- measurement. Ackoff [Ackoff, 1962] provides a refined definition of measurement:

> It (measurement) is a way of obtaining symbols to represent the properties of objects, events, or states, which symbols have the same relevant relationship to each other as do the things which are represented.

Under this definition, a series of symbol sets must be designed, each set of symbols representing a different measurement according to its nature. Numbers are the most commonly used symbols. Other examples are adjectives in natural language, music notation, X-Y plotted coordinates, etc. Stevens [Stevens, 1959] classifies the scales of measurement into nominal, ordinal, interval, and ratio for determination of equality, determination of relative magnitude (greater or less), determination of differences, and determination of ratios, respectively. While all four scales can be used to abstract quantitative data in target systems, qualitative scales represent only nominal or ordinal. Paradoxically, qualitative description is often more informative than quantitative description to designers. An instance is when the temperature of water is at its boiling point (qualitative) versus 212 degrees Fahrenheit (quantitative). The qualitative information has direct meaning, while quantitative data requires the designer to perform further interpretation from his own knowledge and experience.

Qualitative Reasoning

Researchers studying qualitative reasoning about physical systems have provided some means to build qualitative models on top of quantitative properties and equations [de Kleer and Brown, 1984][Forbus, 1984][Kuipers, 1986]. In de Kleer and Brown's qualitative model based on confluences, the qualitative values a variable can have are ***ranks***, representing disjoint abutting intervals that cover the entire number line. The intervals are identified by a series of ***landmarks***. Kalagnanam et. al. [Kalagnanam, et. al., 1991] underpin their observations on qualitative reasoning with some mathematical bases, described by ordinal relations and differential equations. Struss examines the limitations of qualitative reasoning approaches and formalizes a theory of mapping information between quantity and quality [Struss, 1987].

Ambiguity exists in processes that represent qualitative values on rank order scales, but manipulate qualitative equations as if they had had properties on interval scales. The Q1 algebra in the MINIMA system invented by Williams [Williams, 1988] combines real and sign algebra to allow selections of abstractions intermediate between qualitative and quantitative algebras. Generalizing Allen's temporal interval relations [Allen, 1983], Davis applies the 13 possible order relations and the interval operations to any measure space [Davis, 1990]. All the schemes in this group deal with quality in terms of sign, inequality, ordering relations, functional relations, and influences.

Fuzzy Set Theory and Linguistic Variables

While researchers in qualitative reasoning are struggling with ambiguities and attempting to match their qualitative models with the supposed behaviors of quantitative models, another direction is emerging that uses Fuzzy Set Theory [Zadeh, 1965] and linguistic variables [Zadeh, 1973] to describe uncertain landmarks and degrees of membership. The introduction of Fuzzy Theory adds a new dimension to reasoning, especially in the treatment of subjectivity and uncertainty -- the model builder can adjust the landmarks at will with membership functions covering the landmarks and their neighbors. Not only does it provide the mechanism to describe the quality of a numerical interval, Fuzzy Theory also makes the projection of qualitative values to quantity spaces possible. In other words, it bridges the mapping gap between ordinal and interval scales.

Coyne et. al. [Coyne, et. al., 1990] introduce Fuzzy Set Theory and its operations (intersection, union, complement, etc.) to deal with imprecision problems in design reasoning. D'Ambrosio [D'Ambrosio, 1989] extends qualitative perturbation analysis with fuzzy linguistic variables. Fishwick applies the fuzzy number concept to dynamical system simulation [Fishwick, 1990] and suggests that not only state variable values, parameter values, inputs and outputs, but also model and algorithmic structure can be made fuzzy. In this paper, the concentration is on quantitative and qualitative scale mapping with a few discussions on operations, leaving out locality and propagation issues.

Quantity Space

In quantity space, a point, represented by a real number on a continuous scale, describes the current value of an attribute variable. As illustrated by the 212 degrees boiling temperature example, the numerical value may sometimes be misinterpreted by the model builder or other users if the scale and measuring unit is not defined explicitly, or commonly agreed upon. Some people use the Fahrenheit scale to measure temperature, while others use centigrade. To worsen the case, a compounded measuring unit (mass in $\textit{pound-second}^2/\textit{inch}$ produced by Newtonian Law $m = F/a$, for example) can be forgotten some time after the model is build. Thus, a useful solution typically, is to include the measuring unit in the quantitative variables. Quantities are represented be (n, m), where n is a real number and m belongs to a set of measuring units M. A unit conversion table for the elements in set

M can be constructed so that the conversions can be done automatically by the computer.

Quality Space

The quality space described here has three landmarks -- a high limit, a low limit, and a neutral point. In describing water temperature, for example, one might use *as hot as the boiling point* for the high limit, *as cold as the freezing point* for the low limit, and *room temperature* for the neutral point. In judging a design, one might use linguistic modifiers such as *good*, *bad*, and *so-so* for the high limit, the low limit, and the neutral point, respectively. The quality space is separated into segments, as in Figure 1(a), by these three distinct points. Segments such as *above-high-limit* (out of range above the high limit), *positive direction* (between the high limit and the neutral point), *negative direction* (between the low limit and the neutral point), and *below-low-limit* can then be identified. Two linguistic modifiers ***very*** and ***somewhat*** are used to augment the descriptions. In the good-bad judgment example, the seven ranks are *very good*, *good*, *somewhat good*, *so-so*, *somewhat bad*, *bad*, and *very bad*. Figure 1(b) demonstrates these ranks in the quality space.

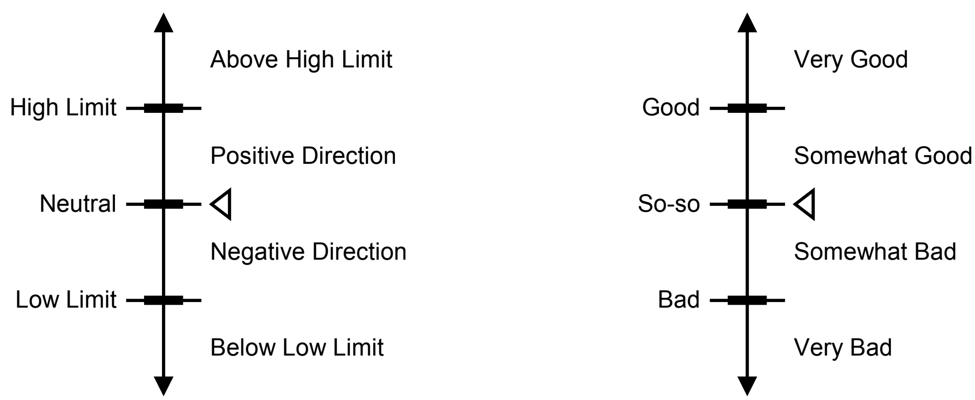

Figure 1: (a) Segments in a Quality Space, (b) Ranks for Good-Bad Judgment

Conceptually, the seven rank qualitative scale is fuzzy because of the subjective, imprecise, and uncertain nature of qualitative attributes. Borrowed from the MYCIN heuristic programming project [Buchanan and Shortliffe, 1984], a confidence factor from the scale 0.0 to 1.0 is associated with each fuzzy qualitative rank[1]. To summarize, a qualitative value is represented by a two-tuple (***r***, ***cf***), where ***r*** is the ***rank*** and ***cf*** is the ***confidence factor***. The confidence factor may be interpreted as the confidence of a judgment, or the assumed probability of the occurrence of a specific rank, depending upon the nature of the attribute variables.

[1] The two terms ***certainty factor*** and ***confidence factor*** are used interchangeably in many expert systems. However, they are not interchangeable in this paper because a ***certainty level*** will be introduced in the next section with a special meaning.

Mapping between Quantity and Quality

The intent of this paper is to devise a scheme, based on threshold concepts and Fuzzy Set Theory, to deal with the uncertainty issues embedded in the mapping of quantity to quality and vice versa.

The Fuzzy Threshold and Its Compatibility Function

A ***threshold*** sets up a conceptual boundary between distinct phases; the ***high***, ***neutral***, and ***low limits*** are treated as thresholds by the scheme. These thresholds, or landmarks in ***qualitative reasoning*** terminology, cannot be satisfactorily represented by a single point on the numerical scale. A model builder might specify 250 pounds to be the high limit -- heavy; however, this high limit may not be so certain if people ask *What about 250.4 pounds, or 248.9 pounds?* As this example indicates, the thresholds are not effectively points, but intervals on the numerical scale. It is more reasonable to map the rank orders in the qualitative description to ranges in the quantity space than to jump directly from order scale to ratios.

In order to generate a valid numerical range for a qualitative rank, the ***compatibility function*** (pulse function which generates bell curves in Fuzzy Set Theory) is employed [Eshragh and Mamdani, 1981]. Figure 2 shows a bell curve and its related parameters. The coordinate system is composed of a ***U*** axis horizontally and a ***Compatibility Factor*** (***CF***) axis vertically.

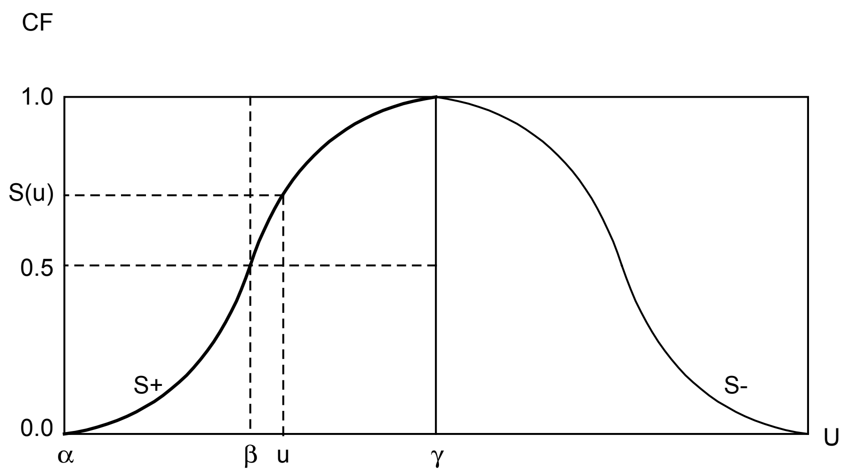

Figure 2: Generation of a Bell Curve and its Related Parameters

The curve, from α to γ, is called an ***S+*** type for its monotonically increasing feature. This ***S*** curve is defined mathematically as follows:

$$S(u;\alpha,\beta,\gamma)=\begin{cases}0 & u=\alpha\\ 2\times\left(\dfrac{u-\alpha}{\gamma-\alpha}\right)^2 & \alpha<u\leq\beta\\ 1-2\times\left(\dfrac{u-\gamma}{\gamma-\alpha}\right)^2 & \beta\leq u<\gamma\\ 1 & u=\gamma\end{cases}$$

In this ***S*** function, $\boldsymbol{\alpha}$ is at the bottom of the curve where the ***CF*** value is 0.0 and $\boldsymbol{\gamma}$ is at the peak point where ***CF*** has the value 1.0. The parameter $\boldsymbol{\beta}$ is the ***crossover*** point; the slope of the tangent line is increasing before $\boldsymbol{\beta}$ but is decreasing after it. In the original ***S*** curve generation, $\boldsymbol{\beta}$ is midway between $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, and its ***CF*** yields the value 0.5. The parameter ***u*** stands for any point within the range $\boldsymbol{\alpha}$ to $\boldsymbol{\gamma}$.

S curves are used to measure compatibility factors for ranks in Fuzzy Theory; they define the neighborhood around each rank. The use of the ***S*** function in this scheme is different, however, in that the curves are applied to bridge qualitative and quantitative values. The bottom of an ***S*** curve rests between two adjacent thresholds with two exceptions -- the two curves at the extreme ranks *above-high-threshold* and *below-low-threshold*. Furthermore, the vertical axis is assigned to a ***confidence factor*** (***cf***, described earlier in **The Quality Space**) instead of a ***compatibility factor*** in order to unify the terms. Figure 3 shows the fuzzy curves generated between a threshold and a neutral point. The left ***S-*** curve (because it is monotonically decreasing) is generated with $\boldsymbol{\gamma}$ (the peak) at the threshold point while the right ***S+*** curve has its $\boldsymbol{\gamma}$ at the neutral point. In both cases, the crossover point $\boldsymbol{\beta}$ is away from the two peaks by a quarter of the distance between the threshold and the neutral point. The value of each point in the curve is the ***confidence factor*** to be associated with its numerical value (***u***) in the qualitative rank.

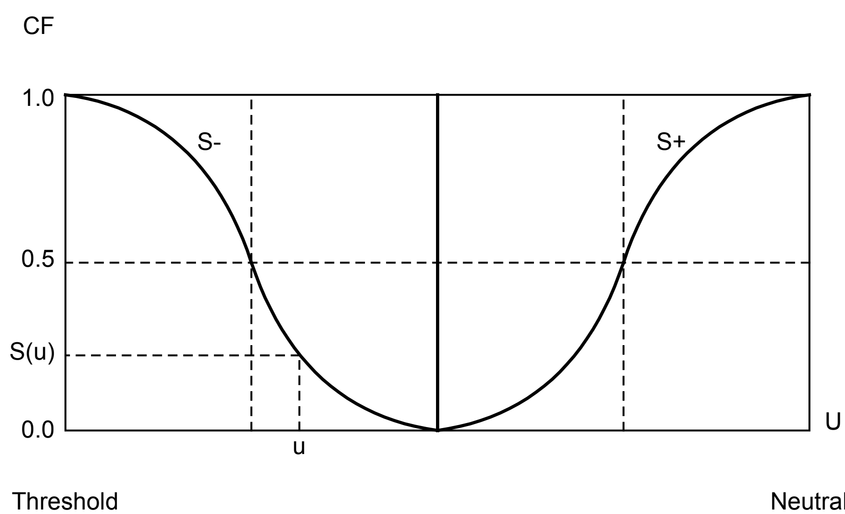

Figure 3: Fuzzy Curves between a Threshold and a Neutral Point

Traveling along the curve in Figure 3, as the numerical value passes from the threshold point to the neutral point, the confidence factor for its numerical value decreases from 1.0 at the threshold rank to 0.0 in the middle and then increases to 1.0 again at the neutral rank. To deal directly with the *somewhat-in-between* rank, a complement curve can be generated so that, from left to right, the confidence factor for the *somewhat-in-between* rank changes from 0.0 to 1.0 and then back to 0.0. Figure 4 depicts the numerical ranges map to quality space where the confidence factors (***cf***'s) are always greater than 0.5. Explanation follows.

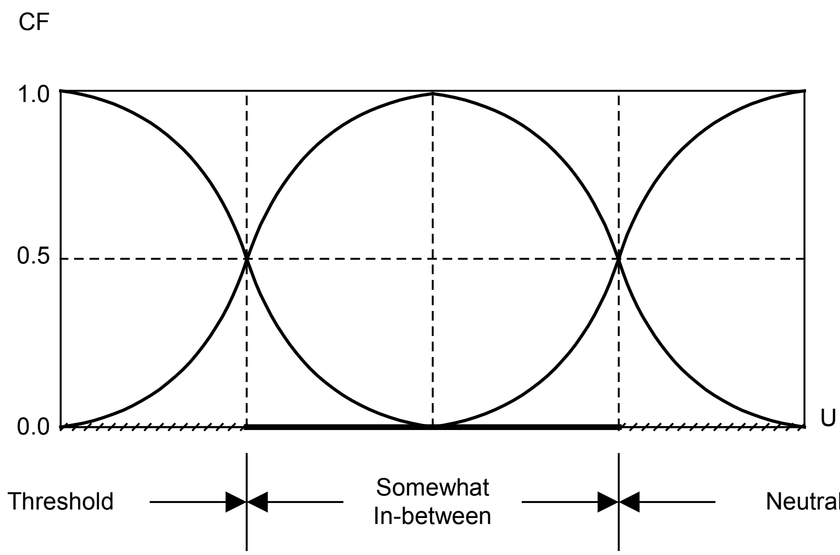

Figure 4: Possible Ranges for Different Ranks

Although the confidence factor varies from 0.0 to 1.0, to be significant, a value for a qualitative variable should have a confidence factor greater than or equal to 0.5 because another rank should be considered if the confidence factor is below 0.5. In other words, any confidence factor below 0.5 can exist only when that specific rank is considered without comparison to other ranks, that is, when the scale is nominal. Because mapping between quantity and quality involves a rank and its adjacent ranks, a limit with a confidence factor 0.5 has to be set; the minimum confidence factor for successfully mapping between the two spaces is 0.5. For example, a qualitative value (*good*, 0.3) is valid only when good is considered as a nominal value without consideration for its relationship with other ranks such as *bad*, *so-so*, *somewhat good*, etc. In the quantity and quality mapping scheme, the value (*good*, 0.3) indicates two other possibilities -- (*very good*, 0.7) or (*somewhat good*, 0.7).

Certainty Level

Although the model builder can adjust the positions of the thresholds on the numerical scale, and each threshold covers an interval of numbers with varying confidence factors, this scheme is weak in that all intervals are equal in length and are evenly distributed between two thresholds. For example, when a model builder specifies 250 pounds for heavy and 150 ponds for the normal weight of a person, he might be more certain about the 250 pound threshold than the 150 pound normal weight. This means that the model builder might want a narrower range for heavy weights and a wider one for normal weights.

To refine the mapping process, a ***certainty level*** is added, defined as the firmness of a specified threshold as judged on a scale 1 to 7 (depending on the context). Each curve for a certainty level is calculated by a ***concentration function*** (a squaring function) from the previous level curve, as shown in Figure 5. Certainty level 1, the most dilated version, is the original bell curve described earlier. In the previous example, if the high threshold is set to be a design constraint -- that is, no consideration is to be given for a *more-than-heavy* person to be a user of the product -- the certainty level of the high threshold would be the highest: 7.

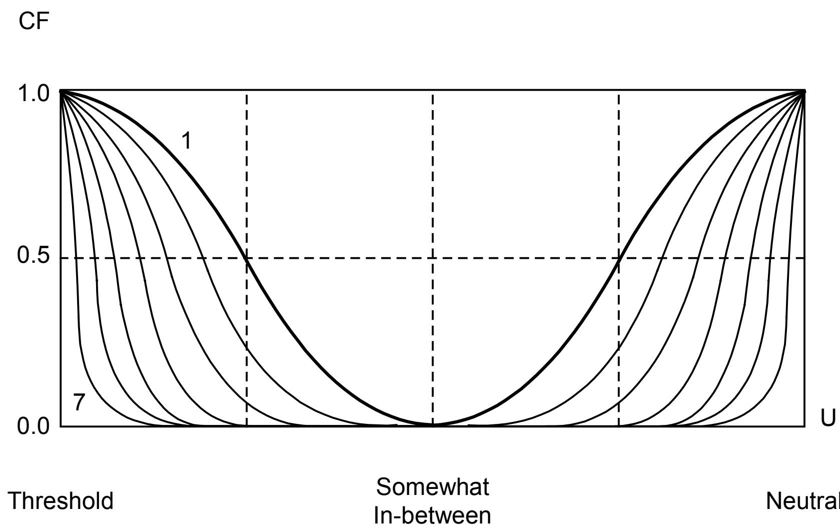

Figure 5: Fuzzy Curves for Seven Certainty Levels

Allowance Factor

One more concept, an ***allowance factor***, defined as the complement of a ***confidence factor***, is necessary to enable the range of a qualitative rank to be calculated. It establishes an imaginary horizontal line cutting through the fuzzy curves. Projecting the intersection points to the quantity scale (the horizontal axis), the mapping scheme, as fully developed in Figure 6, allocates a range in the quantity space (shaded) for each rank. These ranges are more formally defined as ***selection ranges*** in the mapping scheme because they can help the user to select simulation samples. The nomenclature of the quantity-quality mapping scheme is depicted in Figure 6. The model builder can specify a ***minimum allowance factor*** for each threshold so that the tolerance for that threshold can be retained as his original intent. The minimum allowance factor, becoming a constant once specified by the model builder, sets up the ceiling for the confidence factor, which may have changing values during a simulation run. Also, the allowance factors for high and low thresholds and the neutral point can be varied to make the mapping scheme more flexible. The maximum value for an allowance factor is 0.5, for the same reason that minimum confidence factor is 0.5.

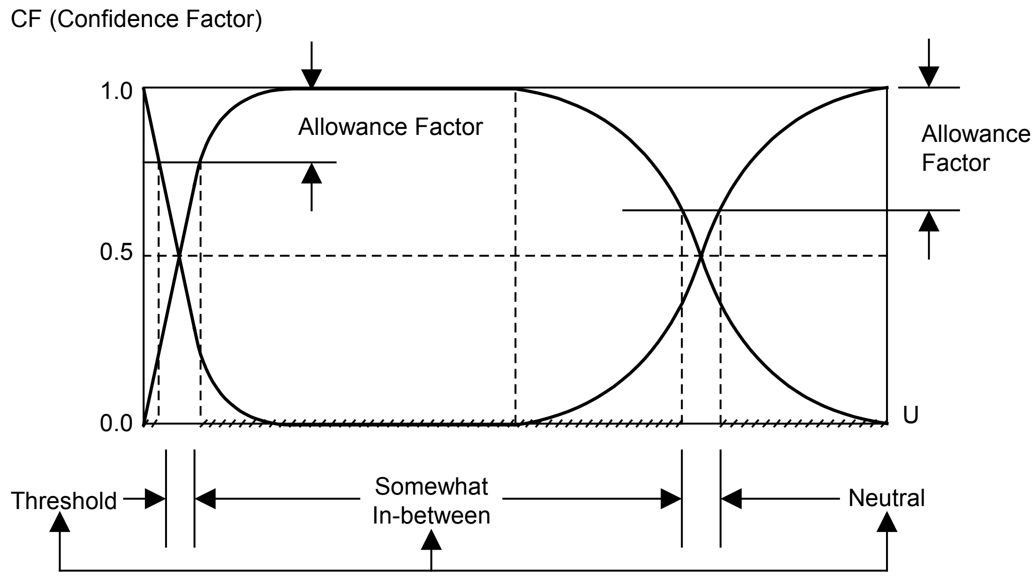

Figure 6: Nomenclature for the Quantity-Quality Mapping Scheme

The two exceptional ranks -- *above-high-threshold* and *below-low-threshold* complete their curves with complements taken symmetrically around the threshold point. Figure 7 shows the details for these special cases.

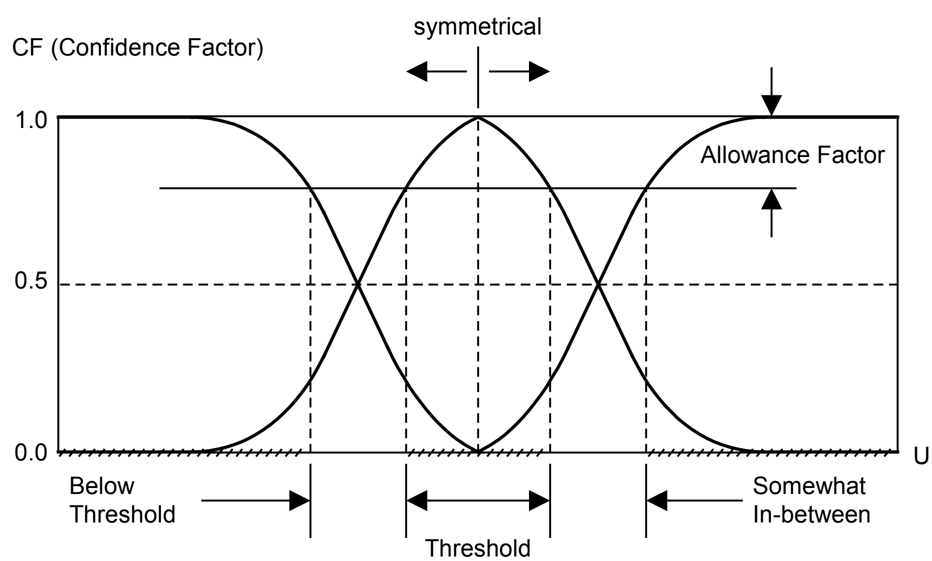

Figure 7: The Fuzzy Curve and Selection Range for Threshold and Below (or Above) Threshold Conditions

The Mapping Mechanism

When a quantitative value is given, the mapping scheme automatically finds the appropriate rank and its confidence factor. When a qualitative values given in (***r***, ***cf***) pair, the confidence factor (***cf***) is first examined to make sure that it is greater than or equal to 0.5 for successful mapping. The scheme then checks to see if the allowance factor ($1 - cf$) is less than the minimum allowance factor the model builder has specified; it uses the minimum allowance factor if the comparison returns true (in other words, the user has expressed a confidence factor that is too high, in the model builder's opinion). Next, the selection range is calculated according to the threshold values, certainty levels, and allowance factors. Once the selection range is allotted, a specific quantitative value for a sample can either be obtained by a random number generator or selected by the user. In summary, rank-order qualities are mapped to disjoint intervals on the numerical scale and the effect is localized; that is, only two adjacent qualities are taken into account at a time. The result is heterogeneous distribution of quality intervals on the continuous numerical scale.

Qualitative Operations

In this section, some operations that can be applied to qualitative variables are discussed. A more rigorous set of operations is still under development.

Qualitative Reasoning and Simulation

The signs of a qualitative variable and its derivatives are the primary concern in qualitative reasoning about physical systems. One landmark with zero value separates the quality space into positive and negative directions. Operations in qualitative simulation may be described in a state table, if-then rules, differentiable functions and equations, etc. The qualitative data type proposed in this paper is, in fact, a super-set of the data abstraction in such a single landmark system and, thus, much more flexible. It is not, however, as flexible as a freely defined multiple landmark system, which is much harder to comprehend in modeling and to control in simulation. All the qualitative operations to be discussed can be applied to qualitative variables in the scheme just proposed.

Fuzzy Logic Operations

There are five major fuzzy logic operations -- fuzzy AND (f_{AND}), fuzzy OR (f_{OR}), probability AND (p_{AND}), probability OR (p_{OR}), and fuzzy negation (f_{NOT}). These operations can be defined by ordinary arithmetic functions, where ***a*** and ***b*** are the confidence factors of two qualities ***A*** and ***B***:

$$a \; f_{AND} \; b = \min(a, b)$$

$$a \; f_{OR} \; b = \max(a, b)$$

$$a \; p_{AND} \; b = a \times b$$

$$a \; p_{OR} \; b = a + b - (a \times b)$$

$$f_{NOT} \; a = 1.0 - a$$

The use of fuzzy or probability operations is context dependent. A general rule is: when the facts *A* and *B* are mutually exclusive, select the fuzzy AND and OR operations; if they are somehow overlapped, use probability AND and OR. A typical example in medical diagnosis concerns two symptoms ***A*** and ***B***, which may contribute to a disease. Suppose symptom ***A*** indicates a 70 percent chance of the disease and symptom ***B*** indicates a 40 percent chance. The likelihood of the disease would be calculated as

$$0.4 \ f_{OR} \ 0.7 \ = \ \max(0.4, 0.7) \ = \ 0.7$$

Whereas the accumulation of evidence can only be considered by using probability OR:

$$0.4 \ p_{OR} \ 0.7 \ = \ 0.4 + 0.7 - (0.4 \times 0.7) \ = \ 0.82$$

Acting as weighting factors, the confidence factors in fuzzy logic operations complement the two-valued (yes or no) logic -- which leads to an interesting point in the discussion of qualitative and quantitative data modeling. The facts confidence factors are associated with are nominal, not measured on ordinal, interval, or ratio scales. A fact is a fact; it carries with it no information for how other facts are defined. Although the confidence factor of symptom ***A*** is 0.7, this does not mean that the chance for the disease not to occur is 0.3; the confidence factor of each nominal value must be individually defined. Also, no comparison of two nominal values should be made -- there are no ordinal ranks, no interval ranges, to say nothing of points on a ratio scale.

Thus, the fuzzy logic operations can be applied to the qualitative variables in the proposed scheme under two constraints: when the definition of quality is nominal and the nominal values imply the same fact. In the symptom-disease example, the values are all true, indicating the occurrence of the disease, but not the negative viewpoints. When a qualitative variable has possible ranks on an ordinal scale, or the ranks will be further mapped to quantitative intervals, the fuzzy logic operations begin to fail and the confidence factor must be greater than or equal to 0.5 in order to validate the ranks. Otherwise, the value of the variable is undecided as explained earlier.

Fuzzy Quality Consolidation

Multiple qualitative variables with different ranks may have to be consolidated to obtain a conclusion. If several judges are evaluating one design, but each of them has his own opinion about how good (or bad) it is, a reasonable means to consolidate their judgments should be used. A ***fuzzy quality consolidation function*** which takes multiple qualitative inputs in (***r, cf***) format and produces a qualitative output, also in (***r, cf***) format can be formulated. Again, because the consolidation function involves different ranks in a quality space, the ***cf*** value is required to be greater than or equal to 0.5. Integer values from negative three (-3) to positive three (+3) are assigned to the seven qualitative ranks -- from *below-low-threshold* (-3) to *above-high-threshold* (+3). Given ***n*** input values (r_1, cf_1), (r_2, cf_2), (r_3, cf_3), ..., (r_n, cf_n), the consolidation function is defined mathematically as

$$CR(ConsolidatedRank) = \frac{\sum_{i=1}^{n} (r_i \times cf_i)}{\sum_{i=1}^{n} cf_i}$$

$$CCF(ConsolidatedConfidenceFactor) = 1 - \frac{\sum_{i=1}^{n} \left| r_i - CR \right| \times cf_i}{(3-(-3)) \times \sum_{i=1}^{n} cf_i}$$

The new consolidated rank (a floating-point number) is then rounded to the closest rank except that the exact mid-values belong to rank segments, not thresholds: −2.5 become −3; −1.5, and −0.5 become −1; +0.5 and +1.5 become +1; and +2.5 becomes +3, as shown in Figure 8. The consolidated confidence factor takes the absolute distance between each sample and the average rank (while it was in floating-point format) into account. The longer the distance, the less the confidence factor will be. In fact, the confidence factor can be treated as an agreement factor for multiple variables. A few examples demonstrate the idea of fuzzy quality consolidation. Inputs (−3, 1.0) and (+3, 1.0) yield the result (0, 0.5). Inputs (−1, 1.0) and (+1, 1.0) yield (0, 0.83333). When the values agree in the same rank, the consolidated confidence factor jumps to 1.0 no matter how much the input confidence factors are because the distances among the samples are all zero. A normal example would be to consolidate (+2, 0.75), (−1, 0.84), and (0, 0.9) which yields (0, 0.82581).

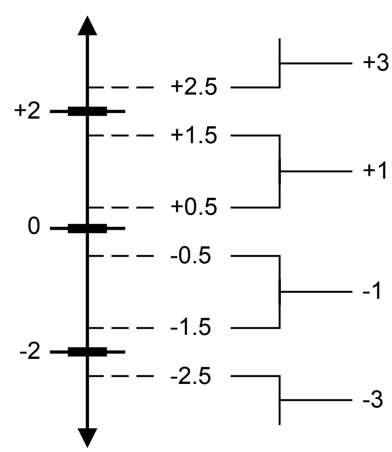

Figure 8: Rounding Exceptions in Fuzzy Quality Consolidation

Conclusion

The quantity-quality mapping scheme described in this paper helps designers set up hybrid models to cope with discrete, imprecise, and uncertain information as well as precise numerical values. Intuitive, qualitative, and subjective judgments, values designers face constantly, can be recorded, integrated and used by others in a group work environment. Although it was originally developed as a tool for designing artifacts, representing ideas and testing new concepts, this quantity-quality mapping scheme has broad potential for use in many fields.

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